A Possible Method of Generating Submillimeter Wave

For the last few years there has been considerable effort to generate, transmit and detect ultramicrowave energy. This effort has yielded substantial progress, but as yet, there has been no break through on the problem. In this brief communication, we would like to introduce a possible method of microwave frequency multiplication. Consider a microwave signal of frequency ω which is made to propagate through a medium whose dielectric constant is a prescribed function of time. The electromagnetic wave interacts with this variable dielectric constant in such a way that its frequency at the output of the medium is $A_f\omega$ where A_f is the frequency multiplication factor of the medium. The principle of operation is illustrated in Fig. 1. Let the electron density of the interaction region D be controlled by the signal e_g in such a way that the dielectric constant of this region is a function of time and is of the form illustrated in Fig. 2. A fourier's series expansion of this waveform is

$$\epsilon = \epsilon_0 \left[\gamma + \frac{m}{2} - \frac{m}{\pi} \sin \omega_m t - \frac{m}{2\pi} \sin 2\omega_m t \cdots \right] \quad (1)$$

or

$$\epsilon = \epsilon_0 \left(\frac{m + 2\gamma}{2} \right) \left[1 - \frac{2m}{\pi(m + 2\gamma)} \sin \omega_m t - \frac{m}{\pi(m + 2\gamma)} \sin 2\omega_m t \cdots \right], \quad (2)$$

where $\epsilon_0\gamma$ is the dielectric constant of the interaction region for $m=0$, ω_m is the fluctuation frequency and $m\epsilon_0$ is the maximum fluctuating amplitude.

If the interaction region D is such that its relative dielectric permitivity is given by[1]

$$K = 1 + \sum_N \frac{Ne^2/m}{\omega_r^2}, \quad (3)$$

where N is the number of electrons per unit volume and

m = the mass of the electron
e = the charge of an electron
ω_r = the natural resonance frequency of an electron,

then in (2)

$$1 \gg \frac{2m}{\pi(m + 2\gamma)} \sin \omega_m t + \frac{m}{\pi(m + 2\gamma)} \sin 2\omega_m t \cdots \quad (4)$$

and

$$\epsilon^{1/2} = \epsilon_0^{1/2} \left(\frac{m + 2\gamma}{2} \right)^{1/2} \cdot \left[1 - \frac{m}{\pi(m + 2\gamma)} \sin \omega_m t - \frac{m}{2\pi(m + 2\gamma)} \sin 2\omega_m t \right] \quad (5)$$

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[1] A. Von Hippel "Dielectrics and Waves," John Wiley and Sons, Inc., New York, N. Y., 1954. See p. 103.

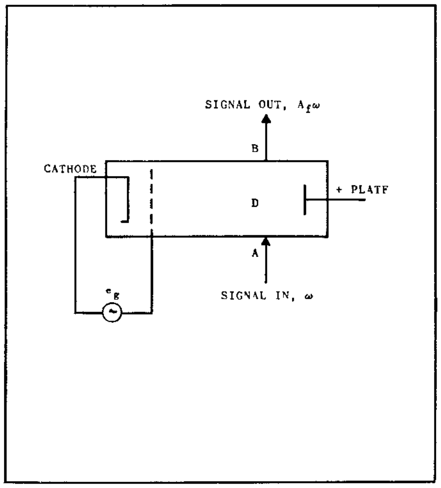

Fig. 1—The principle of operation.

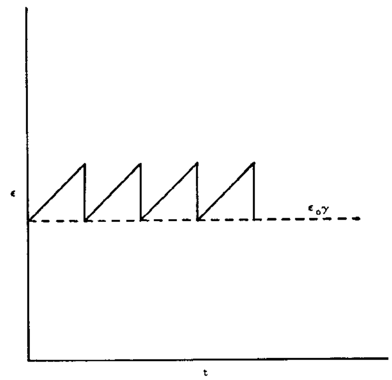

Fig. 2—The effective dielectric constant of the interaction region as a function of time, t.

or

$$\epsilon^{1/2} = \epsilon_0^{1/2} \left(\frac{m + 2\gamma}{2} \right)^{1/2} - b[\sin \omega_m t + \tfrac{1}{2} \sin 2\omega_m t + \cdots], \quad (6)$$

where

$$b = \frac{m\epsilon_0^{1/2}}{\sqrt{2}\pi(m + 2\gamma)^{1/2}}.$$

Now if a signal $Ae^{ikz - i\omega t}$ is applied at the terminal A, and if the propagation time between A and B is so small that the electron density during this time is almost constant, then it can be shown that the signal arriving at B may be represented by

$$e = A \exp [izg - ih(\sin \omega_m t + \tfrac{1}{2} \sin 2\omega_m t + \cdots) - i\omega t], \quad (7)$$

where

$$g = \omega\sqrt{\mu\epsilon_0} \left(\frac{m + 2\gamma}{2} \right)^{1/2}$$

and

$$h = \omega\sqrt{\mu}\, zb. \quad (8)$$

By taking the real part of (7), it can be shown that the amplitude constant of the various harmonic contents of the output signal is a product of zero and higher order Bessel functions of argument h. By a proper choice of h, the amplitude constant of the desired harmonic may be optimised.

It it is desired to obtain an output signal of frequency not necessarily an integral multiple of the input frequency, one might follows a different procedure and write (7) as

$$e = A \exp [iza\sqrt{\mu}\, \omega] \exp [-i\omega(1 + z\sqrt{\mu}\, ab)t - iz\omega\sqrt{\mu}\, adt^3] \cdots \quad (9)$$

where

$$a = \left[\epsilon_0 \left(\frac{m + 2\gamma}{2} \right) \right]^{1/2}$$

$$b = \frac{2m\omega_m}{\pi(m + 2\gamma)}$$

and

$$d = \frac{5m\omega_m^3}{\pi(m + 2\gamma)3}. \quad (10)$$

If the frequency $z\omega\sqrt{\mu}adt^2$ and higher order terms in t are filtered out, the output signal is

$$e = A \exp [-i\omega(1 + z\sqrt{\mu}\, ab)t] \quad (11)$$

or

$$e = Ae^{-i\omega' t} \quad (12)$$

where

$$\omega' = \omega(1 + z\sqrt{\mu}\, ab). \quad (13)$$

The quantity $(1 + z\sqrt{\mu}\, ab)$ may be defined as the frequency multiplication factor A_f.

By substituting the proper values of a and b into (13), it can be shown that this multiplication factor is a function of the modulating frequency f_m. As an example, for the values

$$m = 0.5$$
$$\gamma = 1.25$$
$$\omega_m = \pi 10^9$$
$$z = 1,$$

a multiplication factor A_f of 2.66 is obtained.

For the same set of values, but for $\omega_m = 2\pi 10^9$, we have $A_f = 3.66$. It is seen that the multiplication factor increases with an increase of the modulation frequency and the largest possible multiplication factor depends upon the largest possible modulation frequency. The maximum modulation frequency depends upon the relaxation time[2]

$$\tau = \frac{6m\pi c}{\mu_0 e^2 \omega_0^2}, \quad (14)$$

where

ω_0 = the resonance frequency of an electron
c = the velocity of light.

In practice one must use a modulation frequency f_m such that

$$f_m \ll \frac{1}{\tau}. \quad (15)$$

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[2] *Ibid.*, see also p. 101.